

Influence from the Future*

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ABSTRACT

We argue that some of the parameters in the laws of Nature would be well understood, under the assumption that there is an influence from the future as well as from the past, in the sense that the principle of locality (or causality) is not valid at the fundamental level. However, locality is supposed to be broken only in the mild way that all the non-local influence comes from integrals over the whole of space-time (including the past as well as the future) and acts at all moments and all places with the same effect. Thus the observable effects of the lack of locality can only be seen in the constants of Nature. Our cleanest prediction is the Higgs boson mass being $M_H = 149 \pm 26$ GeV, which in addition assumes that the pure Standard Model is valid (i. e. no new physics) until the Planck scale. Adding the assumption that the two vacuum states needed in our model come about naturally requires a strong first order transition between them; together with the assumption of the Planck units being fundamental, this leads us to also predict the mass of the top quark, $M_t = 173 \pm 5$ GeV, and a more precise value of the Higgs boson mass, $M_H = 135 \pm 9$ GeV. With the assumption of our favourite gauge group beyond the Standard Model: the anti-grand unified group SMG³ (three copies of the Standard Model gauge group, one for each generation, first appearing close to the Planck scale), it is also possible to obtain impressively good predictions for the three fine structure constants in the Standard Model. We also discuss the extension of the SMG³ group by an extra abelian factor $U(1)_f$ and the fermion mass matrices in this model. Other vacua than the present one might appear in the future due to human activity. It is even suggested that life itself could be the miracle needed to avoid a time machine paradox like the matricide paradox: life and we ourselves were made in order to produce a vacuum bomb inaugurating the next vacuum. The breaking of locality is really of the same mild character as suggested by baby universe theory, so the present work may be considered as a development of baby universe theory.

1 Introduction

In the formalism of general relativity there is no obvious obstruction to the appearance of wormholes and baby universes [1], or other topological configurations that easily cause communication of a type incompatible with the validity of a causality or locality principle. It may be technically difficult and even difficult for fundamental reasons—namely the need for negative energy density—to produce large scale wormholes. However at the Planck scale $\sim 10^{-33}$ cm there can be huge quantum fluctuations and even the topology of space-time is expected to fluctuate, so it is hard to see how the locality threatening space-time structures could be prevented. Given quantum mechanics and gravity, it seems almost impossible to construct a theory without space-time foam and non-locality. In baby universe theory it is speculated that the typical such locality violating configuration, namely a handle connected to a weakly curved (relative to the handle size) four dimensional space-time, is to be integrated over all possible “points” of attachment for the two ends of the handle. From this integration, it follows that the effect of the handles (i. e. of the baby universe exchanges) are the same all over space-time. It is understandable that the effect is, in this way, smeared out to be the same all over space-time, from the fact that a baby universe leaving a smooth universe is prevented from taking energy or momentum with it. In Einstein’s theory of general relativity, the information about energy and momentum is to a large extent stored “safely” in the gravitational field far away from where the energy or momentum is really situated.

* Presented by H. B. Nielsen; the manuscript has evolved appreciably since the talk was given.

Heisenberg's uncertainty relation may then be used to argue that, since the leaving baby universe does not take away any 4-momentum, the probability amplitude for its place of departure, i. e. the "point" of attachment of the handle, is the same all over space-time. In this way the non-local effect becomes of the mild type that one can never switch off and must be the same all over space-time. Thus, after integrating out the baby universes, the only residual effect of the non-locality is to change the coupling constants in the low energy effective Lagrangian density. In this way it is not so easy to exclude, on empirical grounds, the existence of such non-local effects.

In the present work it is actually our point to argue that the observed values of the coupling constants support the belief that non-local effects do exist and that, indeed, the future does influence us via such baby universe exchanges (or perhaps simply due to a fundamental lack of locality). If, in the spirit of the project of Random Dynamics [2], one would like to "derive" all the known laws of Nature in some limit from an essentially random fundamental dynamics, then one should indeed at some level investigate the possibility that the principle of locality is not valid fundamentally, but rather to be hopefully derived in some limit. Anyway the main subject of the present article is to use what we believe is a likely consequence of the just suggested baby universe-like mild form of locality-breaking, namely the multiple point criticality principle [3], roughly meaning that the vacuum has many essentially degenerate phases. It is the fine-tuning of the various parameters of the Standard Model, needed to achieve such degeneracies of the different vacuum states, that makes up the messages from the future, as well as the past, and which we want to claim supports the picture sketched above of mild locality breaking. We shall see below that, if our prediction of the Standard Model Higgs boson mass of 149 ± 26 GeV turns out to be correct experimentally, it would be an especially clean test of our claim and it is of course a true *prediction*.

In section 2 we discuss the Standard Model effective Higgs potential and the two main conditions we impose on it, in order to obtain our predictions for both the top quark and Higgs boson masses. The motivation for these two conditions is provided by our assumption that there is a breakdown of locality at the Planck scale. Non-locality and the resolution of the time machine paradox, by requiring the presence of two different vacua in different space-time regions, are discussed in section 3.

In section 4 we argue that it is only likely that the time machine paradox works provided the non-local effect, sensitive to the vacuum being one or the other, is quite strong. This suggests that the vacuum expectation values for the Higgs field, in the two degenerate phases, deviate from one another by an amount of the order of unity in Planck scale units. Our calculation of the predictions of the top quark mass and Higgs boson mass, from this requirement, is then presented in section 5. In section 6 a slightly different, but presumably basically equivalent, formulation of the model is presented, using an ice-water microcanonical ensemble as an analogue, thereby showing that our ideas are not without a kind of precedence in Physics.

In section 7 we briefly describe other work using the same multiple point criticality principle, together with the assumption of the anti-grand unified gauge group SMG^3 , or perhaps the closely related group $SMG^3 \times U(1)_f$, leading to fine-tuned values of the fine structure constants. In fact the three gauge couplings (fine structure constants) in the Standard Model, α_1 , α_2 , and α_3 , are successfully predicted to around ten percent accuracy at the Planck scale. In section 8 we discuss how the extra chiral gauge quantum numbers of the $SMG^3 \times U(1)_f$ model can be used to obtain a realistic fermion mass and mixing hierarchy. Finally, in section 9, we give our conclusion, summary and consolation.

2 Effective Higgs Potential $V_{eff}(\phi)$

It is well-known that in a renormalisable quantum field theory the bare (classical) potential should only have terms involving field factors up to mass dimension four (in four space-time dimensions) and thus, in the Standard Model, must be of the form:

$$V(\phi) = \frac{1}{2}m_{0H}^2|\phi|^2 + \frac{1}{8}\lambda_0|\phi|^4 \quad (1)$$

Since the Higgs field ϕ has $U(1)$ weak hypercharge as well as being a weak isospin doublet, it must occur in the combination $|\phi|^2$. However, when quantum fluctuations of fields are taken into account and one asks for the energy density in a state with a prescribed average value of the Higgs field (in a prescribed gauge), one cannot simply use the bare potential formula eq. (1), but rather the effective potential [4] given to the first approximation (with one-loop corrections) by:

$$V_{eff}(\phi) = \frac{1}{2}m_{0H}^2|\phi|^2 + \frac{1}{8}\lambda_0|\phi|^4 - \frac{1}{2}\text{Tr} \log [\delta^2 S / \delta\phi\delta\phi (\delta^2 S / \delta\phi\delta\phi|_{\phi=0})^{-1}] - \text{Tr} \log [\text{for other fields}] \quad (2)$$

where S is the classical action. The effective potential $V_{eff}(\phi)$ may really be thought of as taking into account the energy density due to zero point fluctuations in the Higgs field ϕ around its central (= average) value, as well as how the zero point fluctuation energy density of the other fields vary as a function of ϕ ; this is at least so for static field configurations. If, as we shall actually assume, the effective potential, calculated according to this formula eq. (2), has

two minima, at $\phi_{\min 1}$ and $\phi_{\min 2}$, with a hill in between, a prescribed average value for the field ϕ in the interval of such a hill would (to get minimum energy density) be realized as an inhomogeneous mixed state. The Higgs field ϕ in this mixed state would take on different values $\phi(x) \simeq \phi_{\min 1}$ and $\phi(x) \simeq \phi_{\min 2}$ in various regions of space-time, rather than by having the field closely fluctuating around a single ϕ -value in the hill interval. This means that if the effective potential is defined precisely as the the minimal energy density obtainable in a state, subject to the constraint of having a prescribed average value for ϕ , then the expression in eq. (2) is not correct for $\phi_{\min 1} < |\phi| < \phi_{\min 2}$. With this precise definition, the effective potential will always be a convex function. However, under conditions where the tunnelling from one such mentioned minimum at $\phi_{\min 1}$ to the other minimum at $\phi_{\min 2}$ is strongly suppressed, the effective potential, as defined in eq. (2), plays an important role for times small compared to the large tunnelling time. Therefore, in the present article, we shall use the not necessarily convex expression eq. (2)—or rather the more accurately calculated expression discussed below—for V_{eff} , ignoring the possibility of averaging over fields which take on wildly different values in different regions. In the following sections, we shall discuss this “averaging” in a more “physical” way, talking about vacuum bombs, the future and destiny of humanity etc., rather than just hiding it in a formal definition of the effective potential.

One can, of course, improve the lowest order (one-loop) effective potential eq. (2), by using higher order corrections. An especially efficient method of improving V_{eff} is to make use of the running coupling constants—by far the most relevant for our work is the Higgs field self-interaction coupling constant $\lambda(\mu)$ —as calculated by integrating up the renormalisation group equations for the various couplings in the Standard Model. A large part of the trace-log corrections in the one-loop approximation formula eq. (2) can actually be included, by just using the form of the bare potential eq. (1), but inserting the running coupling constants as coefficients, identifying the scale parameter μ with the field value $|\phi|$. Taking into account the difference between the bare and the running (= renormalised) couplings in eq. (2), one can obtain the renormalisation group improved effective potential. But just using the running coupling(s) in the bare form of the potential is already quite good, and essentially what we shall use in our own discussion. Only for the purposes of high accuracy calculations is the true renormalisation group improved one loop effective potential really needed.

That is to say a rather good approximation is already achieved by just taking:

$$V_{eff}(\phi) = \frac{1}{2}m_H^2(\mu = |\phi|)|\phi|^2 + \frac{1}{8}\lambda(\mu = |\phi|)|\phi|^4 \quad (3)$$

The concept of a running Higgs mass squared $m_H^2(\mu)$ may only be reasonably defined by excluding the quadratic divergences as causing any scale change. However, the term with the Higgs mass squared coefficient becomes unimportant for Higgs fields large compared to the weak scale, and that is where we have our main interest. The running $\lambda(\mu)$ is easily computed by means of the (first order) renormalisation group equations:

$$16\pi^2 \frac{d\lambda}{d\ln\mu} = 12\lambda^2 + 3(4g_t^2 - 3g_2^2 - g_1^2)\lambda + \frac{9}{4}g_2^4 + \frac{3}{2}g_2^2g_1^2 + \frac{3}{4}g_1^4 - 12g_t^4 \quad (4)$$

Here the $g_i(\mu)$ are the three Standard Model running gauge coupling constants, discussed further in section 7, and $g_t(\mu)$ is the top quark running Yukawa coupling constant, which satisfies the renormalisation group equation:

$$16\pi^2 \frac{dg_t}{d\ln\mu} = g_t \left(\frac{9}{2}g_t^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{12}g_1^2 \right) \quad (5)$$

Because the top quark Yukawa coupling is of order unity, while the other Yukawa couplings are very small, it is only the top quark Yukawa coupling that is significant for the renormalisation group development of the Higgs self-coupling. So we ignored the other quark and lepton Yukawa couplings, including transition Yukawa couplings (quark mixing angles). For large values of $|\phi|$ the effective potential will approximately be given as $V_{eff}(\phi) \approx \frac{\lambda(|\phi|)}{8}|\phi|^4$, where $\lambda(|\phi|)$ is determined by solving the differential equations (4) and (5). In practice we used the second order renormalisation group equations.

With the bare potential form eq. (1), it is only possible to have two minima as a function of $|\phi|$ by having one of them at $|\phi| = 0$. However, for sufficiently large g_t , it is possible to arrange the renormalisation group improved effective potential to have two minima, both of which correspond to strictly positive values of $|\phi|$. With the well-known vacuum expectation value of the Higgs field, $\langle \phi \rangle = 246$ GeV, the top quark Yukawa coupling constant must have a value corresponding to a top quark mass of about 90 GeV or above, in order to obtain two minima with strictly positive $|\phi|$ (norms). If there are indeed two minima, for a top quark mass larger than 90 GeV, the realistic picture is that the minimum corresponding to the experimentally observed properties of weak interaction physics is the smaller $|\phi|$ -valued one, $\phi_{\min 1}$. The Higgs boson mass that will be observed is, of course, what we could call a renormalised

Figure 1: Symbolic illustration of the two main requirements on the effective potential $V_{eff}(\phi)$ which we observe leads to acceptable numerical values for the top quark and Higgs boson masses: 1) Two equally deep minima, 2) achieved for $|\phi|$ values differing, order of magnitudewise, by unity in Planck units.

Higgs mass and is very closely, but not completely exactly, given by the second derivative of the effective potential at $\phi_{\min 1} = 246$ GeV. It is the second derivative in the radial direction that is to be used here. The point of the further small correction to obtain the true Higgs pole mass is that the second derivative of V_{eff} really gives what one might call the Higgs mass squared when the four momentum of the Higgs boson is zero, but the on mass-shell condition does not of course correspond to precisely zero four momentum. This correction is quite calculable and not big.

It is reasonable to think of the free parameters, corresponding to the relevant bare couplings g_{0t} , m_{0H}^2 , λ_0 and the three Standard Model fine structure constants, as to be given by fixing:

A) the renormalised fine structure constants $\alpha_i = g_i^2/(4\pi)$ and the value of the Higgs field norm $|\phi|$ for the phase in which we live, $\phi_{\min 1} = 246$ GeV; these parameters are already very well-known experimentally—the latter vacuum expectation value being essentially an expression for the Fermi-constant of weak interactions;

B) the two less well-known parameters represented by the Higgs boson and top quark masses (with $\langle \phi \rangle$ given, the Higgs boson mass is essentially related to λ while the top quark mass is related to g_t).

The main results presented here are based on the following numerical observation. If the parameters listed under A are fixed by experiment and the two remaining parameters—the Higgs boson and top quark masses—are adjusted so that the effective Higgs potential gets two equally deep minima, the second of which has a Higgs field vacuum expectation value of the order of the Planck scale, then the required masses are quite realistic from an experimental point of view. The needed top quark mass turns out to be 173 GeV, accidentally just the value first estimated by the CDF experiment at FNAL. The corresponding Higgs boson mass is 135 GeV and is at least consistent with all present bounds, as well as with indirect estimates from LEP measurements and radiative corrections. Figure 1 illustrates the two features which, we thus claim, are quite compatible with experiment and the pure Standard Model being valid up to the Planck scale M_{Planck} .

We now wish to show that our idea of the existence of two Standard Model vacua, having the same energy density and with the difference in their Higgs field vacuum expectation values being of order unity in fundamental (Planck) units, can be motivated by the consideration of non-local effects. So, in the next two sections, we take up

our assumption that the otherwise so well-established principle of locality is actually fundamentally violated!

3 Vacuum Bomb and Time Machine

The major assumption in our work—in addition to believing in the pure Standard Model essentially all the way to the Planck scale—is that the principle of locality (or causality) is broken at the fundamental level! This may at first seem a very strong, and even obviously wrong, assumption. However, except at very short distances of the order of the Planck scale, we assume that it is *only broken in the mild way that there is an effect which is the same at all points in the space-time manifold, and that this effect depends on an averaging over all space-time*. This translational invariant non-local effect is much less easy to reveal empirically, if it existed, than an effect that could be switched on and off. It will namely be conceived of as a modification of the parameters in the laws of Nature, the “coupling constants” we may call them, and our non-locality in this mild form only means that the coupling constants depend on what has happened in the past and what will happen in the future (in the sense of an average over all of space-time). Since it is hard to know “what the coupling constants should have been” without such non-locality effects, it is hard to see that there was any modification of them due to effects from the past and the future. It is, however, not a priori impossible, since it could happen that the non-local effects leave such a characteristic signal in the pattern of coupling constants, that we would become (psychologically and scientifically) convinced of the existence of such a non-local mechanism. In fact that is what we want to say: With only very few extra assumptions, the effects of non-locality predict the Higgs particle mass to be 135 GeV, the top quark mass to be 173 GeV, the running Standard Model fine structure constants at the Planck scale correct within 10%, zero cosmological constant etc. ! We in fact, to a large extent, see such a pattern and so perhaps the reader really ought to believe that locality is broken in the mild way we suggest.

It should be remarked that this mild way of breaking locality—and even the idea of breaking locality—is taken over from baby universe theory. Baby universe theory [1] is the study of the effect of small handles—exchange of baby universes—on an otherwise smooth space-time manifold. Since the attachment points on the smoother space-time manifold are integrated over, each handle provides effects which are the same all over the smoother manifold. That is to say the baby universes precisely provide breaking of locality only in the mild way we just described, giving the same effect all over space-time. You may thus consider our non-locality a (slight) generalization of baby universe theory. We consider it a generalization, because we do not exclude the possibility that the fundamental laws of Nature lack the principle of locality fundamentally and not just due to some baby universes popping up, in an otherwise a priori local looking theory of quantum gravity. Locality plays a somewhat roundabout rôle in baby universe theory: In quantum gravity one would work with a Lagrangian that is required to be local, in the sense of being an integral over four dimensional space-time of a Lagrangian density only depending on finite order derivatives of fields at the point in question (similarly string theory is described by a local action). Then baby universes appear and locality gets really broken, but now—hopefully—it turns out to be so mildly broken that, after all, it can only be seen in the coupling constants, as we described above, and locality is effectively restored.

This picture is a bit strange philosophically: The scientist has put the a priori locality assumption of the string theory, or of the quantum gravity action written as a space-time integral, into the models presumably based on the empirical observation of locality; but the empirical observation of locality is strongly based on the fact that the violation of locality (by baby universes) is only *mild*, i. e. the same everywhere so that it only appears in the coupling constants. The basic input of locality at the very fundamental level is not sufficient! So we might as well say that assuming locality broken quite a priori at the fundamental level is preferable. In the spirit of Random Dynamics one would take locality to be broken fundamentally and indeed we have argued [2] that, assuming general reparameterisation invariance, it can easily turn out only to be broken in the mild way; thus we conclude that we really should never have assumed locality at the fundamental level. Anyway it does not matter in the present work, we just need the breaking in the mild way for some reason or the other.

Now in the previous section we saw that, with an appropriate effective Higgs potential, there is the possibility for several “vacuum states” to exist. The presence of matter, of the type and density which we observe in our Universe today, should be considered as only tiny modifications of the vacuum fields, compared to the magnitude of the field modifications needed to shift from one vacuum to another one; the important events, from the physical point of view, are not whether some country gets a new president or so but rather whether we get a new vacuum. It is the presence of different vacua in different space-time regions which is by far the most important feature in space-time, as it is responsible for the effects of non-locality resulting in contributions to the “coupling constants” (the parameters in the laws of Nature). That is to say: the messages from the future, which we can realistically hope to get in our picture, relate to information about the vacuum in coming epochs. We shall refer to this influence from the vacuum in the future as a type of time machine.

Let us, for simplicity, imagine that we have just the possibility of the two vacua achievable by putting the Standard Model Higgs field expectation value equal to either $\phi_{\min 1}$ or $\phi_{\min 2}$, as in figure 1. In reality we may

believe that, by including other degrees of freedom beyond those of the pure Standard Model or by considering non-perturbative effects on a lattice, we could find many other possible vacua; however that does not matter so much for understanding the principal idea. Let us further suppose that one specific phase—our vacuum, the one we live in now with $\langle \phi \rangle = \phi_{\min 1}$ —came victoriously out of the Big Bang. That is to say it came to be the realized phase all over three-space. If the energy density in the various vacuum-candidates were very different it is almost unavoidably the case that the lowest energy phase would win as the cooling of the Universe proceeded, since the higher energy density phases could relatively easily decay into the lowest energy density phase. However, if the energy differences, for some reason or the other, happened to be small, then it could be that a “vacuum” other than the one of lowest energy density was the most stable at the high temperature prevailing in the time shortly after the Big Bang. Such a “vacuum” could then have pushed the other ones away and, by being metastable, survive for a long time—perhaps even an infinitely long time. This possibility opens up the scenario of a vacuum bomb being constructed and causing the transition of space from the metastable phase into the stable one. The moment when that happens, of course, in principle depends on details of the development of the Universe and on politics etc. if human beings are involved in triggering the transition off. Let us denote this moment of vacuum transition by t_{ignition} ; then the various coupling constants, such as e. g. the bare Higgs mass, will depend in a rather smooth way on this ignition-time t_{ignition} due to non-local effects (in the mild form). But now there is the possibility of a paradox, much like the matricide paradox (this is a paradox that can be reached if one possesses a time machine and uses it for going backward in time to kill one’s own mother while she is still a child; at first there seems to be no possible solution as to what can happen, because all possibilities seem contradictory!). It could very likely happen—with say 50% probability—that the non-local effect is such that a late ignition of the truly stable vacuum leads to a value for the Higgs mass, which would give the stable vacuum a much lower energy density than the metastable one; this in turn would imply that the ignition ought to have taken place very shortly after the temperature of the Universe had fallen sufficiently that the truly stable vacuum was in fact also the one with the lowest free energy. While, on the other hand, an early transition would lead to the high temperature favoured vacuum having such a low energy density that it would actually be the truly stable one (while the other one would have higher energy density and would never get competitive). In this case we have indeed obtained a paradox: If the ignition time t_{ignition} is early in the evolution of the Universe the Higgs mass will be such as to make it late or totally impossible, and if it is late the non-local effects will make it early; a contradiction in each case!

According to Novikov and others [5, 6], in these paradoxical cases with time machines, one should find one or more self-consistent solutions. In the case of billiard balls passing through a wormhole and traversing a closed timelike curve, say, one can indeed find such solutions, although they typically have the character of being *fine-tuned*, such as requiring that one ball just barely touches another (or itself returned from the future); the bullet shot at one’s mother could give a superficial wound only. We can also express this by saying that the existence of time machines would imply miracles!

In figure 2 we have shown the expectation from normal physics that the time of ignition largely speaking increases as the bare Higgs mass squared m_{0H}^2 increases, because with m_{0H}^2 increasing the energy density of the other large $\phi = \phi_{\min 2}$ “vacuum” increases and therefore becomes more difficult to reach from our vacuum. The small graphs under the main abscissa axis show the corresponding development of the effective Higgs potential $V_{\text{eff}}(\phi)$ as a function of m_{0H}^2 , with the two minima of $V_{\text{eff}}(\phi)$ denoted by “us” and “other” respectively. Eventually m_{0H}^2 becomes so large that it is the high $\phi_{\min 2}$ vacuum which is unstable and we shall then of course never find any transition, assuming that our low $\phi_{\min 1}$ vacuum won the competition of coming out of the Big Bang. Thus the ignition time t_{ignition} diverges when the value of m_{0H}^2 is such that the two vacua become degenerate. This is so to speak the normal dependence between the ignition-time t_{ignition} and the bare Higgs mass squared m_{0H}^2 . However, with non-locality there is also a (more mysterious) non-local effect and that could easily, as we suggested, go in the opposite direction, i. e. make m_{0H}^2 a decreasing function of t_{ignition} . Four possible non-locality curves are drawn in figure 2, illustrating what the non-local effect could be. The only self-consistent solution is where the non-locality and normal physics curves intersect. In each of the four cases, the self-consistent solution has a value for the bare Higgs mass squared m_{0H}^2 such that the two vacua are nearly degenerate. That is to say that a “time machine miracle”, of the same sort that makes a billard ball running into a time machine (wormhole) and knocking its “younger” self away from the arranged orbit into the wormhole actually just graze itself so that a consistent development occurs, will also make the time machine for mild lack of locality become consistent. Here the miracle is that the vacua become just degenerate: our so-called multiple point criticality principle. This means that the vacuum in which we live—“our” vacuum—lies on the vacuum stability curve [4, 7] and hence the Higgs mass is determined as a function of the top quark mass. In particular for a top quark mass [8] of 180 ± 12 GeV, it follows from recent calculations of the vacuum stability curve [9, 10, 11] that the Higgs pole mass is predicted to be 149 ± 26 GeV.

Figure 2: The ignition time t_{ignition} for a vacuum bomb and the development of the effective Higgs potential $V_{\text{eff}}(\phi)$ as a function of the bare Higgs mass parameter m_{0H}^2 . The solid line labelled “normal physics” shows the usual dependence of t_{ignition} on m_{0H}^2 . Whereas the other four solid lines illustrate possible strong effects of non-locality. The intersection of a “non-locality” curve with the “normal physics” curve occurs for a value of m_{0H}^2 for which the minima of $V_{\text{eff}}(\phi)$ are approximately degenerate.

4 Likelihood of Mechanism, Strong First Order Transition Requirement

At first one might think there is no restriction on what values, $\phi_{\text{min } 1}$ and $\phi_{\text{min } 2}$, the Higgs field could take in the two minima, but really there is an argument that the difference must be rather “large”. The point is that if the difference, $\phi_{\text{min } 2} - \phi_{\text{min } 1}$, in Higgs field values for the two vacua is small, the effect of say the future acting back on the present, depending on which vacuum is present in the future, will be small too. If that is the case there will very likely not be any time machine paradox in the first place and, thus, also no miraculous solution will be needed. Then the Higgs mass parameter m_{0H}^2 only depends very weakly on the time of ignition of the new vacuum and it is, to first approximation, as if there was no time machine at all.

What now means “large” or “small”, for such an influence of say the future on the past and present etc.? We shall only estimate it in a dimensional way. Since we think that the basis of our model is quantum gravity—baby universes and the like—it is natural to assume that the fundamental units are to be taken as the Planck units, i.e. the units based on Newton’s gravitational constant G , the speed of light c and the Planck constant \hbar : $M_{\text{Planck}} \simeq 10^{19}$ GeV. Then we can from dimensional arguments expect that a “large” effect will appear when the field difference, $\phi_{\text{min } 2} - \phi_{\text{min } 1}$, is large compared to one in the Planck units for that field; this means that the “phase transition” between the two vacua is strongly first order. Since now “our” vacuum has an extremely small Higgs field expectation value, $\phi_{\text{min } 1}$, the difference is roughly the same as the other vacuum field value, $\phi_{\text{min } 2}$. Thus we must have $\phi_{\text{min } 2}$ at least of order M_{Planck} to make the whole scheme work. On the other hand it becomes natural that $\phi_{\text{min } 2}$ is not very much larger than M_{Planck} , when we assume—as we do—that the Planck units are the fundamental ones. Thus we end up suggesting that indeed the second minimum must be at the Planck scale: $\phi_{\text{min } 2} \simeq M_{\text{Planck}}$. It is this assumption that then leads to the top quark mass prediction of 173 GeV, as we shall see in the next section.

Whatever the arguments for the second minimum to be at the Planck scale, it can only be true in order of magnitude, because the very concept of the Planck scale is only defined via dimensional arguments, making sense modulo some factor of the order of unity. Our prediction of the top quark mass can therefore be no more accurate than a calculation with an order of magnitude uncertainty in M_{Planck} . It is remarkably good luck for our calculation

that the dependence on the value used for M_{Planck} turns out to be very weak indeed.

For dimensional reasons, the predicted top quark Yukawa coupling must depend upon the ratio of the weak scale to the Planck scale. Now it is well-known [12, 13, 14] that the Standard Model renormalisation group equations (4) and (5) have an infrared quasi-fixed point, when the top quark Yukawa coupling and the Higgs self-coupling are considered as running, but the fine structure constants as being essentially scale independent. As the renormalization point μ goes down towards the infrared, g_t and λ approach this approximate infrared fixed point exponentially as a function of $\log \mu$. But that means that the approach goes as some power law as a function of μ for reasonably small μ (such that we get dominance by the infrared point, but not so low μ that the approximation in which there is a fixed point is spoiled). The value of the running g_t relevant for our top quark mass prediction is in a regime where this approximate fixed point is a reasonable approximation and is approached from below. Thus the deviation of the top quark mass from the fixed point value obtained in the infra-red limit goes as a power law function of the μ to Planck scale ratio. But then it also goes as a power law as a function of the Planck scale, when the weak scale is considered fixed. We looked at the numerical curves and extracted a series of top quark masses (for given weak scale) as a function of $\phi_{\min 2}$. It turned out numerically, from these correlated top quark mass and $\phi_{\min 2}$ values, that the predicted top quark mass deviates from the quasi-fixed point value, of ca. 230 GeV, by a term going inversely as the 42nd root of the Planck scale (relative to the weak scale).

To some extent this connection with the infrared quasi-fixed point top quark mass takes away some of the great impressiveness of our prediction, because it means that in the very first approximation we just obtain the fixed point value; this of course will be the result in essentially any model having a large desert, so that the weak scale is in the infrared, and an unsuppressed top quark Yukawa coupling. Nonetheless our prediction actually deviates from the fixed point value by about 25%, and agrees with experiment much better than if we had just predicted a value in the neighbourhood of the infrared stable point. Indeed we see that the 42nd root converts an uncertainty of a factor of 10 in the Planck scale to only an uncertainty of $\frac{\ln 10}{42} \times 100\% \simeq 5\%$ in the deviation between the fixed point value and the predicted top quark mass. This corresponds then to a couple of percent uncertainty in the total top quark mass, or of the order of 3 GeV.

5 Calculation of the Higgs and Top Masses

As discussed in section 3, the influence from the future in a non-local theory can easily lead to the requirement that the vacuum should have degenerate phases. This gives the condition that the Standard Model renormalisation group improved effective Higgs potential should take the same value in two minima:

$$V_{eff}(\phi_{\min 1}) = V_{eff}(\phi_{\min 2}) \quad (6)$$

One of the minima corresponds to our vacuum with $\phi_{\min 1} = 246$ GeV and eq. (6) defines the vacuum stability curve. We are interested in the situation when $\phi_{\min 2} \gg \phi_{\min 1}$. In this case the energy density in our vacuum 1 is exceedingly small compared to $\phi_{\min 2}^4$. Also, in order that $\phi_{\min 1} = 246$ GeV, the coefficient of ϕ^2 in the effective Higgs potential has to be of order the electroweak scale. Thus, in the other vacuum 2, the ϕ^4 term will a priori strongly dominate the ϕ^2 term. So we basically get the degeneracy condition eq.(6) to mean that, at the vacuum 2 minimum, the effective coefficient $\lambda(\phi_{\min 2})$ must be zero with high accuracy. At the same ϕ -value the derivative of the effective potential $V_{eff}(\phi)$ should be zero, because it has a minimum there. In the approximation $V_{eff}(\phi) \approx \frac{1}{8}\lambda(\phi)\phi^4$ the derivative of $V_{eff}(\phi)$ with respect to ϕ becomes

$$\frac{dV_{eff}(\phi)}{d\phi}|_{\phi_{\min 2}} = \frac{1}{2}\lambda(\phi)\phi^3 + \frac{1}{8}\frac{d\lambda(\phi)}{d\phi}\phi^4 = \frac{1}{8}\beta_\lambda\phi^3 \quad (7)$$

and thus at the second minimum the beta-function (given to first order by the right hand side of eq. (4))

$$\beta_\lambda = \beta_\lambda(\lambda(\phi), g_t(\phi), g_3(\phi), g_2(\phi), g_1(\phi)) \quad (8)$$

vanishes, as well as $\lambda(\phi)$. Here we used the approximation of the renormalisation group improved effective potential [4], meaning that we used the form of the polynomial classical potential *but* with running coefficients taken at the renormalisation point identified with the field strength ϕ . We also do not distinguish between the field ϕ renormalised, say, at the electroweak scale and the renormalised running field $\phi(t) = \phi\xi(t)$ at another scale $\mu(t) = M_Z \exp(t)$ where $\xi(t) = \exp(-\int_0^t dt' \frac{\gamma}{1-\gamma})$. The reason is that, due to the Planck scale being only used in order of magnitude, we shall get uncertainties of the same order as this correction. In fact the anomalous dimension γ is of the order of 1/100, making the difference at most of the order of our uncertainty.

The degenerate minima condition eq. (6) and the associated vacuum stability curve have been studied for the Standard Model in three recent publications [9, 10, 11]. Their results are slightly different but, within errors, are each

consistent with the linear fit

$$M_H = 135 + 2(M_t - 173) - 4 \frac{\alpha_3 - 0.117}{0.006} \quad (9)$$

to the vacuum stability curve, in GeV units. When this degenerate minima condition eq. (9) is combined with the experimental value [8] of the top quark pole mass, $M_t = 180 \pm 12$ GeV, we obtain a rather clean prediction [15] for the Higgs pole mass:

$$M_H = 149 \pm 26 \text{ GeV} \quad (10)$$

If we now also impose the strong first order transition requirement, discussed in the previous section, which takes the form:

$$\phi_{\min 2} = \mathcal{O}(M_{\text{Planck}}) \quad (11)$$

we no longer need the experimental top quark mass as an input, but rather obtain a prediction for both M_H and M_t . So we impose the conditions $\beta_\lambda = \lambda = 0$ near the Planck scale, $\phi_{\min 2} \simeq M_{\text{Planck}}$, and use the second order renormalisation group equations to evaluate $g_t(\mu)$ and $\lambda(\mu)$ at the electroweak scale $\mu = \phi_{\min 1}$ [15]. A change in the scale of the minimum $\phi_{\min 2}$ by an order of magnitude, from 10^{19} GeV to 10^{18} or 10^{20} GeV, gives a shift in the top quark mass of ca. 2.5 GeV. Since the concept of Planck units only makes physical sense w.r.t. order of magnitudes, this means that we cannot, without new assumptions, get a more accurate prediction than of this order of magnitude of 2.5 GeV uncertainty in M_t and 5 GeV in M_H . The uncertainty at present in the strong fine structure constant $\alpha_3(M_Z) = 0.117 \pm 0.006$ leads to an uncertainty in our predictions of $\sim \pm 2\%$, meaning ± 3.5 GeV in the top quark mass. So our overall result for the top quark mass is $M_t = 173 \pm 5$ GeV. Combining the uncertainty from the Planck scale only being known in order of magnitude and the α_3 uncertainty with the calculational uncertainty in the vacuum stability curve, we get an overall uncertainty in the Higgs boson mass of ± 9 GeV. So our Standard Model criticality prediction for both the top quark and Higgs boson pole masses is:

$$M_t = 173 \pm 4 \text{ GeV} \quad M_H = 135 \pm 9 \text{ GeV} \quad (12)$$

6 The Ice-Water Analogue

As we have just seen, the numerical coincidence is quite impressive—we get very close to the experimental top quark mass!—but the story about the time machine paradox and the miracle may not a priori be too convincing. It is therefore important to explain that the multiple point criticality principle, which is really equivalent to the requirement of degenerate vacua, is rather closely analogous to a phenomenon happening in well-known and well-established physics: We refer to the fine-tuning of the temperature in a microcanonical ensemble, such as a mixture of ice and water in a thermally isolated container.

Instead of arguing in terms of the time machine philosophy a very similar sort of physics, and to tell the truth also one violating the principle of locality, may be formulated in analogy to such a microcanonical ensemble. An action of the non-local type we have used above, and which represents the very mild violation of locality (or causality) needed, is simply a highly nonlinear function of reparameterisation invariant space-time integrals $I_i \stackrel{\text{def}}{=} \int d^4x \sqrt{g(x)} \mathcal{L}_i(x)$ of field functions $\mathcal{L}_i(\phi, \partial\phi, \dots)$ that could, by their symmetry properties, have been Lagrangian densities [2]. Now we are interested in vacua and can thus consider the Feynman path integral with time taken purely imaginary; then the configurations dominating the functional integral should be the ones relevant for the vacua having the lowest energy densities. For our highly nonlinear (non-local) action, the configurations dominating the functional integral would be those with least Euclidean action; in general these would correspond to certain specific combinations of the values for the various I_i . At least it would be very usual to have some significant minima in the non-local action as a function of the space-time integrals—the I_i . It would therefore often be a very good approximation to replace the exponentiated action, $\exp(-S_{nl}(I_1, I_2, \dots, I_n))$, by a product of delta functions and, perhaps, still some extra factor multiplying them.

Remembering that the Euclidean path integral is formally of the same type as a partition function for a statistical mechanical model (e. g. field theory), the appearance of delta functions brings microcanonical ensembles to mind. Indeed we do believe that our (mildly) non-local actions are very likely to be well approximated by some microcanonical ensemble; or rather a generalization of the microcanonical ensemble to an ensemble in which not only the energy has a fixed value (the usual microcanonical ensemble), but also other extensive quantities take on specified values (corresponding to them being fixed by delta functions, under the integration leading to the partition function).

Now there is a well-known mechanism whereby microcanonical ensembles, or generalizations thereof, lead to the appearance of more than one phase and the fine-tuning of an intensive quantity: all very much analogous to the consequences of the mild non-locality for which we have already argued. A very familiar example of a microcanonical ensemble is a certain number of water molecules, in a multilayered plastic bag that provides a thermally isolated system, which has got a well defined and pre-specified amount of energy E . Strictly speaking, for this system E should include

the energy spent in expanding the bag into the atmosphere; so we should really use the enthalpy, $E_{\text{water molecules}} + PV$, where V is the (variable) volume and P is the fixed pressure. Since it is not so practical to calculate directly with a microcanonical ensemble, the standard procedure is to approximate the microcanonical ensemble with a canonical one. This procedure can be formally performed by writing the delta function, specifying the value of the Hamiltonian (really the enthalpy) H as equal to the imposed energy E , as the Fourier transformation of a temperature distribution. That is to say we write the delta function

$$\delta(H - E) = \int d\beta \exp(i\beta(H - E)) \quad (13)$$

and insert this Fourier expansion into the microcanonical partition function

$$Z_{\text{micro},E} = \int d\mathbf{q}d\mathbf{p} \delta(H(\mathbf{q}, \mathbf{p}) - E) \quad (14)$$

Then $Z_{\text{micro},E}$ is expressed as an integral over the single inverse temperature variable $\beta = 1/(kT)$ of a canonical partition function $Z(\beta)$

$$Z_{\text{micro},E} = \int d\beta Z(\beta) \exp(-iE\beta) \quad (15)$$

where

$$Z(\beta) = \int d\mathbf{q}d\mathbf{p} \exp(i\beta H(\mathbf{q}, \mathbf{p})). \quad (16)$$

Now for large systems—say macroscopic systems—the integral (15) will be dominated by a very narrow range of β -values. So, having knowledge of the canonical ensemble, we may determine the value of β around which this very small significant β -interval is centered, by finding that value which gives the prescribed value E for the average $\langle H \rangle_\beta$ in the canonical ensemble. Now, for our example of water molecules, it is well-known that the energy or rather the enthalpy $\langle H \rangle_\beta$ has a θ -function singularity, i. e. a jump as a function of β or, equivalently, of the temperature T . So while it is often possible to find a one-to-one correspondence between E and β —in fact $\langle H \rangle$ is a monotonic function of β —there is an interval of E -values which corresponds to the single β -value at which the jump occurs. In this interval the microcanonical ensemble must in reality represent a system of two phases: in one region of the container there is water, in another region there is ice. The jump, of course, occurs at the freezing temperature and is due to the finite latent heat of fusion. Due to this latent heat, the enthalpy of the water at the freezing point is higher than that of the ice at the same temperature.

The absolutely crucial point for our purpose is that if E is prescribed to be in the gap at the jump discontinuity, then the temperature, or equivalently β , must be equal to the freezing temperature. That is to say a very special temperature is obtained *without fine-tuning* the energy (or enthalpy); but rather only by prescribing it to take a value within the jump discontinuity. The latter, however, does not have to be fixed with any great precision, but can be done just very crudely. So one gets a fine-tuning—namely of the temperature or β —for free, in the sense that nothing has to be fine-tuned at the outset! The fact that in English there is a special word—slush—for the mixture of ice and water which is really a microcanonical ensemble, and that this slush is also used to fix the zero point on the Celsius temperature scale, indicates how easy it must be to fine-tune the temperature by means of such a microcanonical ensemble.

We apply this phenomenon to the analogy in which the extensive variables—the energy in the microcanonical ensemble—correspond to quantities like the I_i (integrals over the space-time manifold), and the intensive quantities—the inverse temperature in the canonical ensemble—are the effective coefficients to the I_i in an action linear in the I_i and thereby a local approximation to the true action; i. e. the intensive quantities are really the coefficients in the action—the coupling constants. Then the remarkable fine-tuning coming for free, in this case, is that the coupling constants get fixed at those values for which there is some first order phase transition. These are precisely the values at which several vacua are degenerate; so this analogy is very useful for us in that it predicts that vacua should be degenerate.

In our analogy the extensive variable I_i corresponds to the Hamiltonian H in a microcanonical ensemble. This is illustrated in figure 3, where we can think of $\langle I_i \rangle$ as taking on a prescribed value $I_{i,\text{fixed}}$ just as the average of the Hamiltonian $\langle H \rangle$ must equal the prescribed energy E . At the phase transition the intensive variable (coupling constant) β takes on the value β_{crit} , corresponding to a range of values $\Delta I_i(\beta_{\text{crit}})$ for the extensive variable.

This analogy throws some light on the reason for believing that the values of the Higgs field in the two degenerate vacua should differ by a large amount. Suppose, for example, that the integral of the Higgs field squared over all space-time is the extensive quantity being fixed (like the energy in the microcanonical ensemble):

$$I_1 = \int d^4x |\phi(x)|^2 \quad (17)$$

Figure 3: Illustration of a graphical computation of the value of the artificially introduced dummy (intensive) variable β . Notice that a whole range of $\langle I_i \rangle$ -values gives the same β -value as a solution, namely the value β_{crit} which corresponds to the inverse temperature at the freezing point in the ice-water analogy

Then in order that it shall be likely that a randomly prescribed value $I_{1, \text{fixed}}$ for this integral should just sit in the gap between the values in the two vacua

$$I_1(\phi_{\min 1}) < I_{1, \text{fixed}} < I_1(\phi_{\min 2}) \quad (18)$$

this gap must be large, meaning that the values of the square of ϕ in the two vacua must differ appreciably. With the Planck scale taken as the fundamental scale w.r.t. which this “appreciableness” is to be judged, this condition becomes

$$\phi_{\min 2} - \phi_{\min 1} \simeq M_{\text{Planck}} \quad (19)$$

Otherwise the whole mechanism will be unlikely to work at all. This is analogous to the fact that if the latent heat of fusion for water is not large (in some sense connected with the expected distribution of the prescribed energy) then it will be very unlikely that slush should form; the energy will usually either be too large so that only water is formed, or too small so that only ice is formed. It is only with a large latent heat and strong first order phase transition that it becomes very likely to find slush. In this analogy we do indeed see that the Planck scale difference between the Higgs field values in the two vacua must be expected, for the idea to be likely to work at all. Thus we see that our model can really predict the top quark mass as well as the Higgs boson mass.

7 Fine Structure Constants, also from Multiple Point Requirement

If we take the speculation seriously that, due to non-locality or for other reasons, the vacuum should occur in more than one version with the same energy density, then it is not necessary for there to be just a couple of degenerate vacua: there could very likely be many. The whole multiple point criticality idea really developed out of earlier work [16] aimed at predicting the three fine structure constants of the Standard Model. This work may, in the light of the above ideas, be interpreted [3] as meaning that we look for degenerate vacua in a model characterized by the following two extra assumptions:

1) There is a fundamental regularization, in the sense that at small distances the laws of physics are truly different from an ideal continuum quantum field theory. We effectively assume that this regularization is provided by a truly existing lattice; so that, for example, the Yang Mills fields are fundamentally represented by link variables taking values in the group, rather than by continuum fields. Really, we hope that the precise way Nature is regularized does not matter for the predictions of the critical couplings for which the different vacua are degenerate. This hope has some support in the work by Laperashvili [17] and from some crude estimates we have made. So we expect the values of the fine structure constants, for which phases with confinement already at the regularization scale coexist with phases having confinement only at much longer distance scales or not at all, to be roughly independent of the precise method of regularization. We also assume, in accordance with our philosophy of taking Planck scale units as the fundamental units, that the effective cut-off is provided by the Planck energy: $\Lambda_{\text{regularization}} = M_{\text{Planck}}$.

2) At the Planck scale, or rather a very moderate factor below it, the Standard Model gauge group is extended, in very much the same way as grand unified SU(5) is often assumed; it is just that we assume another gauge group G , namely what we call anti-grand unification (AGUT) in which G is SMG^3 or $SMG^3 \times U(1)_f$. Here $SMG = S(U(2) \times U(3)) \approx U(1) \times SU(2) \times SU(3)$ is the gauge group of the Standard Model (SMG stands for Standard Model Group). This means that we assume that, close to the Planck scale, there are 36 or 37 types of gauge particle, instead of the 12 of the Standard Model ($\dim(SMG) = 12$), in such a way that there are 12 for each generation. Roughly one can say that each generation gets its own photon, its own W^+ , its own W^- , etc. and then there is (perhaps) an extra $U(1)$ gauge particle (called the $U(1)_f$ particle), which is supposed not to matter so much for deducing the values of the fine-structure constants but is relevant for the fermion mass problem discussed in the next section.

Of course we must also assume that, by means of Higgs fields or some other mechanism, the extended group $SMG^3 \times U(1)_f$ breaks down to the phenomenologically observed Standard Model Group SMG , identified as the diagonal subgroup of the $SMG^3 = SMG \times SMG \times SMG$ group. The diagonal subgroup is the group consisting of triplets (u, u, u) of three identical elements belonging to the abstract group SMG . We then add the assumption that, for some reason, the parameters in the laws of Nature get adjusted in such a way as to make several, or rather as many as possible, vacua become degenerate, in the sense of having the same energy density.

In practice lattice gauge theory calculations are usually made in Euclidean space, i.e. with time taken as purely imaginary. This formulation is well suited to study the vacuum, which is by definition a ground state. If several states of the vacuum with the same energy density occur, it will show up in the Euclidean calculation as a transition point—a phase transition point—as a function of some coupling constant (a fine structure constant say); so that on one side of the transition the vacuum has one structure and on the other side another one. The Euclidean calculation is dominated by the lowest energy density state. Now if there are, say, two vacuum states with very closely similar energy densities, it will happen that for one value of some coupling in the neighborhood (of that value for which the two vacua are exactly degenerate) one or the other of the two vacua will be the truly lowest energy one. Therefore the structure and properties of the Euclidean calculation, say Monte Carlo computations, will make a jump. There will be a phase transition in the Euclidean partition function, at just the value of the parameters for which two vacua are degenerate. If one wants to have several degenerate vacua, it will in the Euclidean calculation correspond to a multiple point at which several phases meet.

Now, in principle, we choose to calculate how the parameters are adjusted in the Euclidean lattice theory, so as to make as many different phases as possible meet for a single set of parameters. There will a priori be some surface in the space of parameters along which this set of phases meet. We take it, as our model, that Nature realizes some point on this surface; then we suggest, supported by some speculative calculations, that the fine structure constants observed in the continuum limit are approximately the same all over this surface. Since we also suppose that such continuum couplings at the phase transition points are roughly independent of the type of regularization used, we should be able to predict values for the fine structure constants that are rather stable with respect to the details of the calculation. These numbers are to be identified with the running fine structure constants, where the renormalisation point is taken as the fundamental (Planck) scale.

In order to perform this program we have first to find out where we can get as many phases as possible to meet. We should, in principle, search in the space of all the possible parameters for the lattice theory with our complicated group $SMG^3 \times U(1)_f$; actually infinitely many parameters because we have effectively assumed that the lattice really exists. For groups such as SU(2) or SU(3), phase diagrams with two parameters already exist in the literature [18], from which one immediately sees that there is a point where three phases meet: one described as being in the “Coulomb phase” at the lattice scale and first confining at longer distances, one that confines already at the lattice scale, and finally one phase which, with respect to the continuum part of the group, is in a similar “Coulomb phase” but confines at the lattice scale with respect to the centre of the group, Z_2 or Z_3 respectively. There is a triple point where the three phases meet for SU(2) or SU(3). We should also remark that, even away from the triple point, the phases may merge into each other so that they are not really separated; but that does not matter for our purposes, since we are just interested in the existence of a multiple point. By choosing the couplings—lattice action parameters—corresponding to this triple point for each of the three SU(3) cross-product factors in the AGUT group SMG^3 and the form of the action so as to have no interaction between the three SU(3) factors, we obtain a set of parameters such that we can achieve any combination of phases for the three SU(3) factors just by infinitesimal shifts of these parameter-values. Thus we can, in this what we could call factorized way, obtain a multiple point for the whole group $SU(3)^3$ where 3^3 phases meet. By analogy, it is also expected that the different types of non-abelian group in the AGUT group can be adjusted independently to reach the multiple point for each type separately; thereby a multiple point for the whole cross product is achieved, with a number of phases meeting equal to the product of the number for each type alone. For the abelian groups, however, we expect that the maximal number of phases meeting will be achieved in a more complicated way. We do not have a completely safe calculation of how many phases can meet when the abelian part

of the group is taken into account, but we estimate that, at least approximately, the meeting of the highest number of phases is achieved in what we call a “hexagonal” scheme. In this scheme, we have terms in the lattice action involving more than one of the $U(1)$ groups in the cross-product at a time. While there is no possibility for an interaction of this type between the gauge fields of non-abelian groups, $SU(2)$ and $SU(3)$, there is such a possibility for the abelian groups: in a continuum formulation for $U(1)$ groups, there is an allowed term in the Lagrangian density of the form $F_{\mu\nu}^{Peter}(x)F_{\mu\nu}^{Paul}(x)$. Since even $F_{\mu\nu}^{Peter}(x)$ is gauge invariant for the abelian case, we are allowed to construct this type of interaction between different $U(1)$ -gauge fields; a corresponding non-abelian term would not be gauge invariant. Here the names *Peter*, *Paul* etc. are used to distinguish the different cross-product factors in G —in this case the $U(1)$ factors.

The “hexagonal” form of the total lattice action for several $U(1)$ groups contains various interaction terms, which are lattice variants of $F_{\mu\nu}^{Peter}(x)F_{\mu\nu}^{Paul}(x)$ or more complicated lattice terms involving several of the $U(1)$ groups, arranged so that the total lattice action has a “hexagonal” symmetry. This symmetry can, very abstractly, be identified with the group of a hexagonally symmetric lattice in a three dimensional space. This symmetry property of the action at the proposed multiple point reflects a group of rotations—or at least linear transformations—in the covering group \mathbf{R}^3 of the group $U(1)^3$ contained in the group SMG^3 . It is this discrete group of rotations (when the action is used to define the metric in the covering group space) which has the same structure as the group of symmetries of a hexagonal lattice in the covering group. By choosing a lattice action to have symmetry like this hexagonal one, the phases transformed into each other under the symmetry operations are forced to meet at the proposed multiple point. In this way one may relatively easily get a lot of phases to meet. We have not investigated carefully, yet, the effect on our results of the extra abelian group $U(1)_f$, but we imagine that it essentially does not mix with the other $U(1)$ factors and that we can ignore it, as far as the diagonal subgroup couplings are concerned.

At the multiple point where most phases meet, as estimated above, we then evaluate the diagonal subgroup couplings. This is done mainly by analytical estimates, using computer generated data taken from the literature on the critical couplings for the groups $SU(3)$, $SU(2)$, and $U(1)$ as input. For the non-abelian groups one gets, in first approximation, the following simple expression for the diagonal subgroup fine structure constant α_{diag} :

$$\frac{1}{\alpha_{diag}} = \frac{1}{\alpha_{Peter}} + \frac{1}{\alpha_{Paul}} + \frac{1}{\alpha_{Maria}} \quad (20)$$

where α_{Peter} , α_{Paul} and α_{Maria} are the fine structure constants for the corresponding three cross-product factors in SMG^3 ; at the proposed multiple point their values are taken to be those at the triple point for a single $SU(2)$ or $SU(3)$ group. For the abelian $U(1)$ groups the story is more complicated and we rather get in first approximation:

$$\frac{1}{\alpha_{1\,diag}} = \frac{6}{\alpha_{crit}} \quad (\text{first approximation}) \quad (21)$$

but with some corrections (due to more complicated terms in the action needed to ensure the multiple point) the factor 6 becomes 6.8.

Finally we obtain our predictions of the fine structure constants at the Planck scale:

$$1/\alpha_1(M_{\text{Planck}}) = 52 \text{ (60)} \pm 5 \quad 1/\alpha_2(M_{\text{Planck}}) = 48 \pm 6 \quad 1/\alpha_3(M_{\text{Planck}}) = 56 \pm 6 \quad (22)$$

where the number in brackets is the predicted value of the $U(1)$ fine structure constant after making the corrections beyond the first approximation referred to above. Extrapolation of these numbers to the the weak interaction scale, using the following Standard Model renormalisation group equations for the gauge coupling constants g_i ,

$$16\pi^2 \frac{dg_i}{d \ln \mu} = b_i g_i^3; \quad \text{where} \quad b_1 = \frac{41}{6}, \quad b_2 = -\frac{19}{6}, \quad b_3 = -7 \quad (23)$$

leads to

$$1/\alpha_1(M_Z) = 96 \text{ (103)} \pm 5 \quad 1/\alpha_2(M_Z) = 32 \pm 6 \quad 1/\alpha_3(M_Z) = 16 \pm 6. \quad (24)$$

There is agreement with the (\overline{MS}) experimental numbers [20]

$$1/\alpha_{1,exp}(M_Z) = 98.29 \pm 0.09 \quad 1/\alpha_{2,exp}(M_Z) = 29.61 \pm 0.06 \quad 1/\alpha_{3,exp}(M_Z) = 8.55 \pm 0.44 \quad (25)$$

well within the theoretical uncertainties, which we have crudely estimated. They include uncertainties from the Monte Carlo data used of the order of 5% of the inverse fine structure constants at the Planck scale. Our several corrections suffer from uncertainties of the same order of magnitude. For the $U(1)$ coupling there is still some uncertainty as to precisely which phases meet at the proposed multiple point; in particular the question as to whether some of the discrete subgroups may or may not confine separately.

8 Quark and Lepton Masses and Mixings compatible with $SMG^3 \times U(1)_f$

Given its success in predicting the values of the Standard Model gauge coupling constants, one might ask whether the SMG^3 model might also provide some understanding of the values of the Standard Model Yukawa coupling constants and the quark-lepton mass problem. Indeed the broken chiral gauge quantum numbers of the quarks and leptons, under the symmetry groups SMG_a ($a=1,2,3$), distinguish between the three generations and have the potential to generate the fermion mass and mixing hierarchy [2]. This hierarchy corresponds to different degrees of suppression for various transitions from right-handed to left-handed fermion states, each of which carry different SMG^3 quantum numbers. The most promising way of explaining these mass suppression factors is in terms of the partial conservation of such chiral flavour quantum numbers [12]. In fact the mass gaps between the three fermion generations are readily explained by the broken SMG^3 gauge quantum numbers. Unfortunately, however, it is not possible to explain all the mass splittings within each generation, such as the ratio of the top and bottom quark masses, in this way. The SMG^3 model inevitably predicts [19]:

$$m_u m_c m_t \simeq m_e m_\mu m_\tau \leq m_d m_s m_b \quad (26)$$

So, together with Gerry Lowe, we were led to extend the gauge group by an extra abelian flavour group factor $U(1)_f$. If we maintain the feature of the anti-grand unified model that the irreducible representations of the Standard Model are not combined into larger irreducible representations of the extended gauge group (intuitively combining irreducible representations together generally invites unwanted mass degeneracies), then the $SMG^3 \times U(1)_f$ model is the only non-trivial anomaly-free extension of SMG^3 with no new fermions. Furthermore the $U(1)_f$ charges are essentially unique:

$$\begin{pmatrix} d_L & u_R & d_R & e_L & e_R \\ s_L & c_R & s_R & \mu_L & \mu_R \\ b_L & t_R & b_R & \tau_L & \tau_R \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1 \\ 0 & -1 & 1 & 0 & 1 \end{pmatrix} \quad (27)$$

We constructed a computer program to search over various mass matrix scenarios, using partially conserved $SMG^3 \times U(1)_f$ gauge quantum numbers, and found that the extended model could naturally accomodate the fermion mass spectrum and mixing angles up to unknown factors of order unity [19].

Very recently—and actually after the meeting on Corfu—we have developed, together with Douglas Smith, the type of fit that the computer seemed to have reached into an explicit representation, as a model in terms of some supposed Higgs fields breaking the group $SMG^3 \times U(1)_f$ down to the Standard Model Group SMG , and finally to $SU(3) \times U(1)$ by the Weinberg-Salam Higgs field ϕ_{WS} . The Higgs field ϕ_{WS} has, in our model, to take highly nontrivial quantum numbers under the group $SMG^3 \times U(1)_f$. Of course for the subgroup of it which is the usual Standard Model Group, the quantum numbers of ϕ_{WS} must be the well-known weak hypercharge $Y/2 = 1/2$, doublet under $SU(2)$ and singlet under $SU(3)$. The Standard Model Higgs field, of course, satisfies the usual charge quantisation rule [2]:

$$\frac{Y}{2} + \frac{1}{2} \text{“duality”} + \frac{1}{3} \text{“triality”} \equiv 0 \pmod{1} \quad (28)$$

The most natural way to incorporate this charge quantisation rule in the $SMG^3 \times U(1)_f$ model is to assume that such a rule holds for each component SMG_a . If we assume that we only have singlet or fundamental matter (fermion and scalar) field representations for all the non-abelian gauge groups $SU(3)_a$ and $SU(2)_a$, the charge quantisation rules analogous to eq.(28) determine the non-abelian representations from the $U(1)_a$ charges $y/2|_a$. So the three weak hypercharges $y/2|_a$ and the $U(1)_f$ flavour charge Q_f together completely specify a matter field representation.

We imagine that the group $SMG^3 \times U(1)_f$ is spontaneously broken down to the Standard Model Group SMG by (essentially) four Higgs field vacuum expectation values (VEVs) called say: S, W, T and ξ in units of the Planck mass. These Higgs fields VEVs respect the usual SMG and hence have weak hypercharges $y/2|_a$ satisfying

$$Y/2 = y/2|_1 + y/2|_2 + y/2|_3 = 0 \quad (29)$$

The biggest vacuum expectation value is taken by the field S with quantum numbers

$$(y/2|_1, y/2|_2, y/2|_3, Q_f)|_S = (1/6, -1/6, 0, -1), \quad (30)$$

but that is supposed to be just of order unity in fundamental scale units, so that matrix elements are in reality not at all suppressed by this VEV, i.e. we take $S = 1$ and act as if the group is totally broken down as far as this field could achieve. Since the Higgs field S gives no suppression, it is hard to detect in a phenomenological fit of the fermion mass spectrum. Consequently there is an ambiguity in guessing the quantum numbers of the other VEVs, namely W, T, and ξ . Instead of proposing for ξ say $(y/2|_1, y/2|_2, y/2|_3, Q_f)|_\xi = (1/6, -1/6, 0, 0)$ we could as well use, instead of ξ itself, a combination of ξ and S and thus declare that really the quantum numbers of ξ were $(y/2|_1, y/2|_2, y/2|_3, Q_f)|_\xi =$

$(0, 0, 0, 1)$. Modulo such extra factors of S we were led to the following proposal for the quantum numbers of the Higgs field expectation values:

$$(y/2|_1, y/2|_2, y/2|_3, Q_f)|_W = (0, -1/2, 1/2, -4/3) \quad (31)$$

$$(y/2|_1, y/2|_2, y/2|_3, Q_f)|_T = (0, -1/6, 1/6, -2/3) \quad (32)$$

$$(y/2|_1, y/2|_2, y/2|_3, Q_f)|_\xi = (1/6, -1/6, 0, 0) \quad (33)$$

The quantum numbers of the Weinberg-Salam Higgs field in our proposal are:

$$(y/2|_1, y/2|_2, y/2|_3, Q_f)|_{\phi_{WS}} = (0, 2/3, -1/6, 1) \quad (34)$$

With these quantum numbers one can evaluate the combination of Higgs fields needed to provide nonzero values for the various matrix elements in the three mass matrices, namely for the quarks with electric charge $Q = 2/3$ called M_u , the one for the quarks with $Q = -1/3$ called M_d and finally for the charged leptons M_l . Strictly speaking there are several possible ways to produce the same quantum numbers, but usually one of those ways is obviously dominating, since the other ones are too suppressed. At least this is true if one uses our fitted values: ($S = 1$), $T \simeq 1/12$, $W \simeq 1/6$, and $\xi \simeq 1/10$. In this way the following quark mass matrix structure was obtained:

$$M_u \simeq \begin{pmatrix} S^\dagger W^\dagger T^2 (\xi^\dagger)^2 & W^\dagger T^2 \xi & (W^\dagger)^2 T \xi \\ S^\dagger W^\dagger T^2 (\xi^\dagger)^3 & W^\dagger T^2 & (W^\dagger)^2 T \\ S^\dagger (\xi^\dagger)^3 & 1 & W^\dagger T^\dagger \end{pmatrix} v \quad M_d \simeq \begin{pmatrix} SW(T^\dagger)^2 \xi^2 & W(T^\dagger)^2 \xi & T^3 \xi \\ SW(T^\dagger)^2 \xi & W(T^\dagger)^2 & T^3 \\ SW^2(T^\dagger)^4 \xi & W^2(T^\dagger)^4 & WT \end{pmatrix} v \quad (35)$$

While for the charged leptons:

$$M_l \simeq \begin{pmatrix} SW(T^\dagger)^2 \xi^2 & W(T^\dagger)^2 (\xi^\dagger)^3 & (S^\dagger)^2 WT^4 \xi^\dagger \\ SW(T^\dagger)^2 \xi^5 & W(T^\dagger)^2 & (S^\dagger)^2 WT^4 \xi^2 \\ S^3 W(T^\dagger)^5 \xi^3 & (W^\dagger)^2 T^4 & WT \end{pmatrix} v \quad (36)$$

Here $v = \langle \phi_{WS} \rangle / \sqrt{2} = 174$ GeV. Unknown coefficients of $\mathcal{O}(1)$ in the matrix elements have been ignored. A three parameter order of magnitude fit [21], using eqs. (35) and (36) with $S = 1$ fixed, successfully reproduces all the observed fermion masses and mixing angles within a factor of two—see table 1.

Table 1: Best fit to experimental data. All masses are running masses at 1 GeV except the top quark mass which is the pole mass.

	m_u	m_d	m_e	m_c	m_s	m_μ	M_t	m_b	m_τ
Fitted	3.8 MeV	7.4 MeV	1.0 MeV	0.83 GeV	415 MeV	103 MeV	187 GeV	7.6 GeV	1.32 GeV
Experimental	4 MeV	9 MeV	0.5 MeV	1.4 GeV	200 MeV	105 MeV	180 GeV	6.3 GeV	1.78 GeV

	V_{us}	V_{cb}	V_{ub}
Fitted	0.18	0.029	0.0030
Experimental	0.22	0.041	0.002 – 0.005

9 Conclusion and Consolation

Based on two or three “basic” assumptions, we have been able to predict an impressive number of parameters in the Standard Model. The assumption which we studied most in the present article is what we have often called the “multiple point criticality principle”; this means that the Universe shall allow for more than one—really as many as possible—vacuum states with (at least approximately) the same energy density. This assumption could be understood in a model in which there is a very mild form of non-locality, namely where the coupling constants are influenced by averages over all space-time (the future as well as the past), and then the degeneracy of the vacua appeared as a miracle needed to make the time machine (of the mild form) be consistent. Our cleanest prediction is that, given the top quark mass from experiment to be $M_t = 180 \pm 12$ GeV, the Higgs boson mass should be $M_H = 149 \pm 26$ GeV. But with the additional assumption that the phase transition should be strongly first order, so that this time machine mechanism should have a good chance of being relevant, we require that the difference in the Higgs field vacuum expectation values between the two phases should be rather big, on the scale of the supposed fundamental

units—Planck units. This leads to the prediction of the top quark mass as $M_t = 173 \pm 5$ GeV and a more precise prediction of the Higgs particle mass as $M_H = 135 \pm 9$ GeV.

To get further predictions it was necessary to include our second basic assumption, namely that there is what is sometimes called anti-grand unification, meaning we introduce the non-simple “unifying” group SMG^3 or its extension to $SMG^3 \times U(1)_f$. Really this means that we assign a separate set of gauge particles—with a mutual interaction structure just identical to that of the set of 12 gauge particles in the Standard Model—for each of the three generations. The $U(1)_f$ group contributes an extra abelian gauge particle, coupling to the right handed particles in the second and third generations in a way allowed by the anomaly cancellation constraints. The latter requirement to a very large extent fixes the $U(1)_f$ charges of all the fermions.

The main motivation for considering this rather specific AGUT gauge group, broken down to the diagonal subgroup of SMG^3 at the Planck scale is, of course, its successful phenomenological predictions. However it originated from some speculative ideas [2], involving the “confusion” mechanism in random dynamics and the observation that the Standard Model Group SMG is very skew—in the sense of having exceptionally few automorphisms. In fact, perhaps most importantly, we can characterise the group $SMG^3 \times U(1)_f$ as the largest possible gauge group beyond the Standard Model, satisfying the following principles:

1. Each gauge field transforms the already known 45 species of Weyl fields (3 generations with 15 in each) nontrivially; thus we only look for a gauge group $G \subset U(45)$, where $U(45)$ here stands for the group of unitary transformations of these known 45 Weyl fields.
2. There should be neither gauge anomalies nor mixed anomalies.
3. The various irreducible representations of Weyl-fields for the Standard Model Group are required *not* to be united under the full group G , but rather to remain as separate irreducible representations even under G .

The predictions of the three fine structure constants in the Standard Model, using the principle of having degenerate vacua and our AGUT gauge group, agree well with experiment—in fact better than our calculational accuracy until now. Finally we put forward a model of the fermion masses, using our favourite gauge group $SMG^3 \times U(1)_f$, and obtained a rather good fit to the *order of magnitudes* of the quark and lepton masses in terms of only three parameters.

All our results are compatible with a single model, in which there is no significant new physics until an order of magnitude below the Planck scale, where one finds the scalar fields connected with our mass matrix fit and the spontaneous breakdown of the AGUT group to the Standard Model Group. Even higher in energy there is some truly existing regularization—but it is speculated not to be so significant which one, even the superstring could be classified as a useful regularization for our purpose—and there will then be very likely lots of new physics. Especially wormholes or baby universes are expected to be present in the space-time foam and cause the mild breaking of locality, which we use in our model to argue for the degenerate vacuum hypothesis. Again the details, of whether it is really baby universes or wormholes that do this job or that locality is just not a fundamental principle of Nature, are unimportant for our predictions.

We note that both a vanishing cosmological constant, $\Lambda_{eff} = 0$, and strong CP conservation, $\Theta_{QCD} = 0$, are characterised as meeting points of phases [22, 23]. Thus mild non-locality, and its affinity for co-existing phases, could also help to explain the cosmological constant problem and the problem of why Θ_{QCD} (and the weak vacuum angle Θ_{weak} which is presumably not practically measurable) should be so small. Then we can claim to have fitted at least order of magnitudewise *all* the parameters of the Standard Model, even including gravity, in our approach. However we must admit to have taken the Standard Model Higgs vacuum expectation value, $\langle \phi \rangle = 246$ GeV, from experiment. Our hope of explaining the gauge hierarchy problem, in terms of a vanishing Higgs mass at a phase boundary [3, 19], fails for the strongly first order phase transition we have argued for in this paper.

If one accepts the success of our fit of the Standard Model parameters as confirming our model, it is in a way sad: its main consequence is that experiments will not find any supersymmetric partners and, on the whole, very little new physics at accessible energy scales. We simply predict a Higgs particle of mass $M_H = 135 \pm 9$ GeV and then essentially a desert up to the Planck scale. The presence of other particles near the electroweak scale would contribute to the renormalisation group beta functions and, in general, would spoil our successful predictions of the top quark mass and the fine structure constants.

Well: there are one or two suggestions for possible new physics in our model, which could just have a chance of being observed:

- The Weinberg-Salam Higgs field in our AGUT scheme actually belongs to a multiplet together with coloured components. Such coloured scalar particles could for symmetry reasons be, like the Weinberg-Salam Higgs field

itself, very light from a Planck scale point of view. Although this does not mean that they must be light enough to be experimentally accessible in the near future, there is at least a chance that they are.

- In our estimates of the fine structure constants, it seems that discrete subgroups of the gauge group play a non-negligible rôle and actually improve the predictions somewhat. Now contrary to the continuum part of the group—the Lie group part so to speak—a discrete group gauge theory does not give rise to any massless particles in general. With our principle of degenerate vacua, however, there is a special chance that some “low energy” physics (compared to the Planck scale) could result: According to our multiple point criticality principle, discrete subgroups are expected to lie on the phase boundary between confining and Coulomb-like degenerate vacua. If for some reason the phase transition between the vacua happened to be second order rather than first order, then the flux strings, which really constitute the physics of discrete gauge theories, would have just zero string tension. The point is that in the confinement phase the electric flux appears as strings, whereas in the Coulomb-like phase the magnetic flux appears as strings. Since both should continuously reappear or disappear at a second order phase transition, the tensions must go to zero there. If this indeed happened, some flux strings from discrete subgroups might be able to contribute forces between particles charged with the discrete charges. This would be with zero tension from the Planck scale point view, again giving a chance of being found experimentally some day. Very likely we would first see such effects as radiative corrections. For instance a vertex could be corrected by two of the involved particles having a flux string connecting them, while they separate after having been locally created from the third particle. Could this possibly be an idea for explaining the deviations from the Standard Model predictions for the $Z^0 \rightarrow b + \bar{b}$ or $Z^0 \rightarrow c + \bar{c}$ decay modes discussed at this meeting? The discrete subgroup involved could easily couple differently to the different quark flavours, because it does not necessarily have to be a subgroup of the Standard Model Group itself but could be e. g. a subgroup of our AGUT group.
- Really our calculations would not be spoiled, if the new physics at low energy did not appreciably disturb the renormalisation group running of the Higgs self coupling, the top Yukawa coupling and the gauge coupling constants. Extra scalar fields would contribute rather weakly to the running of the gauge couplings. Also, given the precedence of the Weinberg-Salam Higgs particle coupling very weakly to many particles, it is not out of the question that some scalar(s) could couple so weakly to the matter fields as to leave our predictions essentially undisturbed.
- Systems of particles decoupled from the ones we know are of course not excluded, but they would be harder to see the closer they are to being completely decoupled.

But, in general, we must admit that the confirmation of our model would mean that there will be no new physics at accelerators on any reasonable time scale!

In this “sad” scenario, one might foresee a future in which research on the fundamental scale forces could only be done by performing high accuracy investigations of the Standard Model parameters and deducing information about details of Planck scale physics. If there was really a good theory, one might learn something from any given amount of information rather independently of whether it was taken at very high energy or with high accuracy at more accessible energies, provided it is not just a general consequence of the low energy tail of the theory. Indeed if our influence from the future picture were true, then it might, to a very weak extent, be possible to see the future through the measurement of the parameters of the Standard Model ! In a way we are already on the way to read the future in the Standard Model parameters, when we propose that our model can be checked by investigating whether the mass of the Weinberg-Salam Higgs particle has the borderline value 135 ± 9 GeV. If indeed it has, we are seeing a bad prophecy for the future in the parameter M_H : we shall all kill ourselves by a vacuum bomb!

But one could imagine that with a deep understanding of the numbers one could—and this is perhaps rather realistic—estimate the degree of difficulty in making such a vacuum bomb, by accurately determining the Higgs mass and thereby the gain in energy per unit volume achievable by a transition to the new vacuum state. The less the gain in energy, the bigger a bubble of new vacuum has to be before it can expand by itself. One would expect that the easier it is to produce the critical bubble size, and thereby the bomb, the easier it will be to get funds for building an accelerator that could achieve it and the sooner will be the end. So a not completely unrealistic attempt to predict something about the future from the Standard Model parameters is actually at hand!

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